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MHD fluid flow through porous medium with periodic permeability

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Abstract

Fully developed flow of a magnetorheological fluid through a non-isotropic porous medium bounded by parallel plates under the effect of an external magnetic field is considered. Permeability is taken as a periodic function of the transverse direction of the fluid flow. The Darcy-Brinkman-Lapwood-Lorentz equation for the fluid flow in porous media has been used and solved under non-slip boundary conditions by Power Series Method and the results validated by the Numerical Shooting Method. Finally, the analysis of results is made of the influence on the velocity, volumetric flow, and wall shear stress taking into account the Ochoa-Tapia and Whitaker's effective viscosity relationship.

Keywords: Hartmann magnetic number; Mathew permeability model; effective viscosity; magnetorheological fluid.

Flujo de fluido MHD a través de un medio poroso de permeabilidad periódica

Resumen

Se ha considerado el flujo totalmente desarrollado de un fluido magneto-reológico a través de un medio poroso no isotrópico delimitado por placas paralelas bajo el efecto de un campo magnético externo. La permeabilidad se toma como una función periódica de la dirección transversal al flujo. La ecuación Darcy-Brinkman-Lapwood-Lorentz para el flujo de fluidos en medios porosos se ha utilizado y resuelto en condiciones de frontera no deslizante por el método de series de potencia y los resultados han sido validados por el método numérico del disparo. Por último, el análisis de los resultados se hace sobre la influencia en la velocidad, flujo volumétrico y el esfuerzo de deformación del fluido en la pared teniendo en cuenta la relación de viscosidad efectiva de Ochoa-Tapia y Whitaker.

Palabras clave: Número magnético de Hartmann; modelo de permeabilidad de Mathew; viscosidad efectiva; fluidomagneto-reológico.

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I. INTRODUCTION

There is a growing interest in studying the behavior of fluid flow in non-homogeneous porous media, whether they are natural or not. Two of the inherent properties that affect fluid transport through porous media are porosity and permeability. These, in turn, could modify fluid properties such as effective viscosity, therefore it is important to develop models of these properties that allow the behavior of fluid flow through porous media to be described and predicted, [1]. Take, for example, the celebrated Brinkman's equation [2], which is the most recognized model of fluid flow through porous media in the presence of macroscopic boundary, (*c.f.* [3] and the references therein) due to its incorporation of a viscous shear term necessary to handle no-slip condition on a macroscopic, solid boundary. Various authors have discussed validity and limitations of Brinkman's equation. Rudraiah [4], suggested that Brinkman's equation is the most appropriate model of flow through porous layers of finite depth, while Parvazinia *et. al.* [5], determined that when Brinkman's equation is used (depending on Darcy number, *Da*) three different flow regimes are obtained: a free flow regime ($Da \geq 1$); a Brinkman regime ($10^{-6} \leq Da < 1$); and a Darcy regime ($Da < 10^{-6}$). Nield [6] , elegantly concluded that the use of Brinkman's viscous shear term requires a redefinition of the porosity near a solid boundary. This underscores the need for variable permeability flow modelling. Sahraoui and Kaviany [7], and Kaviany [8], studied the case of flow through variable permeability media when using Brinkman's equation and emphasized the need for variable permeability near macroscopic boundaries, whether slip or no-slip conditions are applied. Lundgren [9], showed that the effective viscosity depends on the porosity of the medium and the viscosity of the base fluid, and formally proved the validity of the Brinkman equation for dilute concentration of particles.

Although in three-dimensional flow through naturally occurring and heterogeneous media permeability is a tensorial quantity, idealizations of flow through two space dimensions and flow through porous layers have given rise to variable permeability modelling using algebraic functions of one of the space variables. A number of variable permeability models are available in the literature (*cf.* [10], [3], [11] and the references therein). These models have also received considerable attention in the study of magnetohydrodynamic (MHD) flow through porous media. This type of flow has many applications that include power generation, magnetohydrodynamic pumps and accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, oil industry, purification of crude oil and in sprays and aerosols, biotechnology, medicine, optical modulators, tunable optical fiber filters, optical grating, optical switches, stretching of plastic sheets and metallurgy, like other industrial application (*cf.* [12], [13], [14], [15], [16]).

A problem of interest in this field is that of MHD flow between two parallel plates, known as Hartmann flow (*cf.* [17], [18], [19], [20] and the references therein). In the absence of MHD influence, fluid flow through a porous channel bounded by two flat plates has received considerable attention in the literature due to the importance of this configuration in furthering our understanding of flow behaviour through channels (*cf.* [21], [22], [23], [24], [25] and the references therein).

Hassanien *et al.* [26], Hassanien [27], Jang and Chen [28], and Elaiw *et al.* [29] reduced the two dimension permeability model of Chandrasekhara *et al.* [30] at one dimension, in the form of $K=K\infty (1+de^{-y/\gamma})$ where d and γ are constants, and $K\infty$ is the value of permeability at the edge of the boundary layer. Later, other authors employed the two dimension permeability model of Chandrasekhara *et al.* (*cf.* [31], [32], [30], [33]) in the study of mixed convection on boundary layer. As reported by Cheng [10], in soil mechanics the permeability variation may be assumed to be in one dimension only (*cf.* [34], [35] and the references therein). The most widely model used is $K=K_o\,(1+\beta y^*)^{\eta}$, where β and η (typically, $\eta=2$) are parameters of curve fittings, K_o is the characteristic permeability of the medium, and $y^*=y/d$, where d is a characteristic length of geometry. Other permeability models used include:

$K=K_o\,(\cos\beta y^*+\eta\,\sin\beta y^*)^2$, (1)
and,
$K=K_o\,(e^{\beta y^*}+\eta\,\sinh\beta y^*)^2$, (2)

Rees and Pop [36] studied the free convection in a vertical porous medium with the exponential model of permeability $K=K_\infty+(K_w-K_\infty)\,e^{-y/d}$, where K_w is the permeability at the wall, K_∞ is the permeability of the ambient medium, and d is the length scale over which the permeability varies. Additionally, they [36] indicated when a porous medium is bounded by an impermeable surface it is well known that the porosity and hence the permeability increases near that surface. This increase may be due to the channeling effect that occurs in porous media near the boundary.

Alloui *et al.* [37] investigated the natural convection in a porous layer with an exponentially variable permeability as a function of the thickness of the layer, in the form $K=e^{cy}$ where c is a fitting parameter. Abu Zaytoon [38] analyzed the behavior of flow in a porous layer with exponential permeability with the model $K=1/2_e\,(e-e^{-y})$. Choukairy and Bennacer [39] presented a numerical and analytical analysis of the thermosolutal convection in a heterogeneous porous layer enclosed in a rectangular cavity with a permeability changing with the depth, increasing from the bulk to the horizontal surfaces with the model $K(y)=1+4(2y)^n$ where n is a parameter per section of the layer under study.

Pillai *et al.* [40] offered a study of the steady flow of a magnetorheological fluid in an inclined channel over a porous bed with a decaying exponential permeability that depends on the depth using the model $K=K_o\,e^{-cy}$ which was proposed in a previous study by Sinha and Chadda [41]. Narasimha Murthy and Feyen [42], studied the influence of the variable permeability on the two MHD basic flows in porous media, the model of variable permeability studied is $K(y)=K_o\,(1+y/h)^2$, where K_o is the permeability in the interior of the porous medium and h the thickness. Two other important variable permeability models in the study of MHD flow have been reported in the works of Srivastava and Deo [43] and Mathew [44]. Srivastava and Deo [43] employed the variable permeability model K=K_0 (1-εy)^2, where 0≤ε<1, in their study of Poiseuille MHD flow in a channel, while Mathew [44] employed a periodic variable permeability model in the study of two-dimensional MHD convective heat transfer through a porous vertical channel. It is this latter periodic permeability model that is of interest to the current work in which we investigate the fully developed MHD flow through a porous medium bounded by parallel plates. The objective is to shed some light on the effects of the flow and medium parameters (Darcy number, Hartmann number, porosity, and wave amplitude) on the flow characteristics of velocity, flow rate, and wall shear stress, when the permeability variations are due to a periodic function. This is accomplished by obtaining a power series solution to the momentum equation governing the MHD flow through an infinite porous channel between two parallel, horizontal, and impermeable plates. Solution obtained using the currently proposed method is compared with the numerical shooting method for the same problem in order to validate the results obtained. Numerical calculations were performed using *Mathematica®* symbolic software.

II. Mathematical formulation of the problem

Consider the unidirectional, fully developed MHD flow of an incompressible, viscous, electrically conducting fluid through an infinite porous medium channel bounded by two impermeable flat plates, shown in Fig. 1, subjected to an external uniform, transverse magnetic field. Assuming a low Reynolds magnetic number, the external magnetic field does not induce a magnetic field inside the porous medium. It is also assumed that any induced electric field in the porous medium is negligible. Flow in the channel is assumed to be driven by a constant pressure gradient. Variations in permeability across the channel are assumed to be periodic and in accordance with the model introduced in Mathew [44], namely

$$K(\tilde{y}) = \frac{K_o}{1+\varepsilon\, cos\left(\frac{\pi\tilde{y}}{h}\right)}, \qquad (3)$$

wherein h is the thickness of the layer (width of the channel), K_o is the average permeability, and ε is the amplitude of the permeability variation. According to Mathew [44], the amplitude is $\varepsilon<1$.

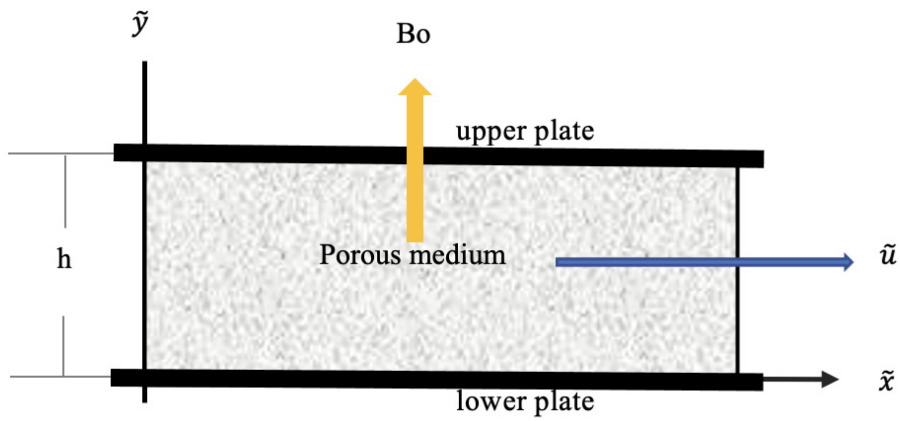

Fig. 1. Representative Sketch. Direction of flow is in the x-direction.

III. Governing equations

Geindreau and Auriault [45] provided comprehensive analysis of MHD flow through porous media, which includes the full set of equations governing the flow. The momentum equation used in their analysis is the one obtained by Rudraiah *et.al* [46] through modification of Brinkman's equation and Darcy's law. This equation is used in the current work (a form of which is listed as equation (5), below).

The governing equations for the fluid flow in the porous medium at hand, are the following continuity and momentum equations, respectively:

$$\nabla \cdot \widetilde{V} = 0, \tag{4}$$

$$\nabla \tilde{p} = \widetilde{F_B} + \mu_e \nabla^2 \widetilde{V} - \frac{\mu}{K(\tilde{y})} \widetilde{V}, \tag{5}$$

where $\tilde{V}$ is the velocity vector field, $\tilde{F_B}$ is the Lorentz body-force, $\tilde{p}$ is the pressure, μ is the the base fluid viscosity, μ_e the effective viscosity in the porous medium, and K is the permeability.

In a two-dimensional domain, we let $\tilde{V} = (\tilde{u}, \tilde{v})$, where $\tilde{u}$ is the velocity component in the $\tilde{x}$-direction and $\tilde{v}$ is the velocity component in the y-direction. For fully-developed, unidirectional flow, $\tilde{v}=0$ and $\partial\tilde{v}/\partial\tilde{y}=0$. Continuity equation (4) thus yields $\partial\tilde{u}/\partial\tilde{x}=0$, or $\tilde{u}=\tilde{u}(\tilde{y})$. Furthermore, we take variations in permeability in the y-direction only and assume that $K=K(\tilde{y})$.

Written in component fully developed fluid flow, (4) and (5) with (3) become, respectively:

$$\tilde{u} = \tilde{u}(\tilde{y}), \tag{6}$$

$$\mu_e \frac{d^2\tilde{u}}{d\tilde{y}^2} - \frac{\mu}{K_o}\left[1 + \varepsilon \cos\left(\frac{\pi \tilde{y}}{h}\right)\right]\tilde{u} - \sigma B_o^2 \tilde{u} = \frac{\partial \tilde{p}}{\partial \tilde{x}}, \tag{7}$$

where,

and,

$$\widetilde{F_B} = \tilde{J} \times \tilde{B} = -\sigma B_o^2 \tilde{u}, \tag{9}$$

$\tilde{J}$ is the current density, $\tilde{B}$ the magnetic field intensity, σ the electrical conductivity of the fluid, $\partial\tilde{p}/\partial\tilde{x}$ the pressure gradient, μ the fluid viscosity, μ_e the effective viscosity in the porous medium, and $\tilde{F_B}$ is the Lorentz body-force.

The non-slip boundary conditions for (7) are:

$$\tilde{u}(0) = 0, \qquad \tilde{u}(h) = 0, \tag{10}$$

To convert (7) into a dimensionless form, the following variables are introduced:

$$y = \frac{\tilde{y}}{h},\ x = \frac{\tilde{x}}{h},\ u = \frac{\tilde{u}}{U},\ Da = \frac{K_o}{h^2},$$
$$M^2 = \frac{\sigma B_o^2 h^2}{\mu},\ \frac{\partial p}{\partial x} = \frac{h^2}{\mu U}\frac{\partial \tilde{p}}{\partial \tilde{x}} = -P \tag{11}$$

where U is the characteristic velocity, M the Hartmann number, Da the Darcy number, and P ($P > 0$) the non-dimensional pressure gradient.

According to Ochoa-Tapia and Whitaker [47], the effective viscosity is:

$$\mu_e = \frac{\mu}{\phi}, \tag{12}$$

where Ø is the average porosity of the porous medium.

The non-dimensional equation, obtained from (7) using (11) and (12), takes the form:

$$\frac{d^2u}{dy^2} - \frac{\phi}{Da}[1 + \varepsilon\, cos(\pi y)]u - \phi M^2 u = -\phi P \tag{13}$$

The boundary conditions associated with (13) take the form:

$$u(0) = 0, \qquad u(1) = 0, \tag{14}$$

The dimensionless volumetric flow rate is given by:

$$Q = \int_0^1 u dy \tag{15}$$

and the dimensionless shear stress is given as:

$$\tau_w = \left.\frac{\partial u}{\partial y}\right|_{y=0,1}, \tag{16}$$

where the subscripts 0 and 1 signify shear stress values at the lower and upper plates, respectively.

IV. Method of solution

Merabet *et al.* [48] indicated that for the case of flow through porous media, as governed by the Darcy-Lapwood-Brinkman model, exact solutions are rare. In some cases, it is possible to find exact solutions, but this requires special functions, such as the Airy's, Bessel or modified Bessel, or the Nield-Kuznetsov functions (*cf.* [10], [49], [50] and the references therein).

For the solution of Eq. (13), we use the quasi-analytic Power Series Method (PSM) to approximate the solution, due to its versatility and speed of convergence, [51]. We thus let:

$$u(y) = \sum_{n=0}^{\infty} a_n y^n = a_0 + a_1 y + a_2 y^2 \ldots, \tag{17}$$

where a_n are coefficients.

Using (17), together with Taylor series expansion for cos(πy), namely

$$\cos(\pi y) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\pi y)^{2n}, \tag{18}$$

in equation (13), we obtain:

$$\sum_{n=2}^{\infty} n(n-1)a_n y^{n-2} - \frac{\phi}{Da}\left[1 + \varepsilon \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\pi y)^{2n}\right]\sum_{n=0}^{\infty} a_n y^n - \phi M^2 \sum_{n=0}^{\infty} a_n y^n = -\phi P\,, \qquad (19)$$

The polynomial obtained in this case for the approximation of Eq. (13) is in the order $O(y^{16})$, and the power series coefficients are:

$$\begin{aligned} a_0 &= 0\,, \\ a_2 &= -\frac{P\phi}{2}, \\ a_3 &= \frac{(1 + DaM^2 + \varepsilon)\phi}{6Da} a_1, \\ a_4 &= -\frac{P(1 + DaM^2 + \varepsilon)\phi^2}{24Da}, \\ a_5 &= \frac{\phi(-3Da\pi^2\varepsilon + Da^2M^4\phi + 2DaM^2(1+\varepsilon)\phi + (1+\varepsilon)^2\phi)}{120Da^2} a_1 \\ a_6 &= -\frac{P\phi^2(-6Da\pi^2\varepsilon + Da^2M^4\phi + 2DaM^2(1+\varepsilon)\phi + (1+\varepsilon)^2\phi)}{720Da^2}, \end{aligned} \qquad (20)$$

and so on....

With the modified boundary conditions (14) the coefficient a_1 is obtained finally.

$$a_1 = \frac{\frac{P\phi}{2} + \frac{P(1 + DaM^2 + \varepsilon)\phi^2}{24Da} - \cdots}{1 + \frac{(1 + DaM^2 + \varepsilon)\phi}{6Da} - \cdots}, \qquad (21)$$

V. Results and discussion

The periodic permeability model used in this work is given in dimensional form by Eq. (3), and has the following dimensionless form:

$$K(y) = \frac{K_o}{1 + \varepsilon\, cos(\pi y)}, \qquad (22)$$

Since $-1 \leq \cos(\pi y) \leq 1$, this permeability function renders non-positive permeability values when $\varepsilon \geq 1$, which could yield unrealistic velocity profiles. We must therefore restrict our analysis to using $\varepsilon < 1$, as suggested by Mathew [44] and seen in Figs. 2, 3, and 4.

Variations of the velocity profile, $u(y)$, with increasing values of ε are illustrated in Table 1, which demonstrates the increase in velocity with decreasing ε in the domain $0<y<0.5$, for chosen l values of other parameters. This is because the permeability increases with decreasing values of ε, as can be seen from Eq. (22). For $0.5<y<1$, Table 1 shows a decrease in the velocity with decreasing values of ε. Again, this reflects the decrease in permeability. At y=0.5, permeability values are independent of ε. Hence, one expects the velocity values to be the same for all values of ε. The discrepancy seen in Table 1 at y=0.5 might be an indication that the value of ε must be less than unity (as anticipated by Mathew [44]), or it might be due to the approximation process used in this work.

Table 1. Comparison of results of u(y) with the variation of ε

M=1, Da=0.1, ϕ=0.35, P=5			
y	**ε=0.00001**	**ε= 0.01**	**ε=1**
0.1	0.05885986575331	0.05881836937986	0.05456989151355
0.2	0.10243691694991	0.10237108164153	0.09543483093448
0.3	0.13241426900880	0.13234965976741	0.12505493761491
0.4	0.14994975596004	0.14991038460802	0.14445556477380
0.5	0.15572065211580	0.15572080925714	0.15340016278921
0.6	0.14994983488785	0.14998952577218	0.15070971235798
0.7	0.13241439825207	0.13247933443023	0.13471453591065
0.8	0.10243704842848	0.10250319884888	0.10385957768043
0.9	0.05885994836826	0.05890169088545	0.05775121214247

The increase in the velocity when the Darcy number Da is increased, for chosen fixed values of all other parameters, is illustrated in Fig. 5. As can be seen, the velocity profile is parabolic and the maximum velocity variations are small for $Da \geq 1$. This might be indicative that $Da=1$ plays an important role in the determination of the Brinkman regime. This seems to be in line with the conclusion that Parvazinia *et. al.* [5] have reached, in that the Brinkman regime is $10^{-6} \leq Da < 1$. Therefore, one can conclude that the Darcy-Lapwood-Brinkman model used in the current work is well-suited in predicting velocity behavior within the choice of parameters used, including the Ochoa-Tapia and Whitaker's equation for effective viscosity.

For fixed values of ε and Da, within the range of the Brinkman model, the increase in the Hartmann magnetic number M results in a corresponding and progressive decrease in velocity, as illustrated in Fig. 6. This is indicative that the increase in magnetic field strength (that is, an increase in the Hartman number) in the transverse direction hinders the flow along the channel, thus decreasing the velocity. This decrease in velocity is quantified in Table 2 which illustrates the values of velocity at the centre of the channel and shows the progressive decrease in velocity at $y=0.5$ with increasing values of M, relative to the velocity value at $M=0$.

Table 2. ***u(y)*** **variation at** $y = 0.5$ **with** M **for fixed values of** ε **= Da = 0.01,** $\phi = 0.35$**, and** $P = 5$**.**

	u **(0.5)**	**Reduction %**
M = 0	0.04460471010648	-
M = 1	0.04422399628337	0.85
M = 3	0.04135843981796	7.28
M = 5	0.03638828984087	18.42

To finish this first part, we study the behavior of velocity as a function of porosity ϕ. The maximum stability value studied is $\phi = 0.45$, where the velocity curve remains parabolic for the rest of the fixed parameters, as can be seen in Fig. 7. For higher porosity values, the behavior of the velocity is unstable, so the Brinkman's model considering the viscosity relationship of Ochoa-Tapia and Whitaker is valid under the fixed parameters here studied.

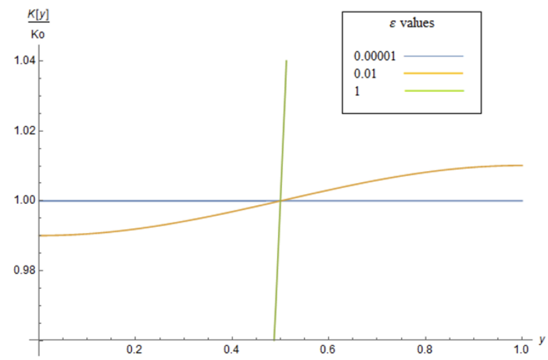

Fig. 2. Plot of $K(y)/K_o$ relationship with values of ε.

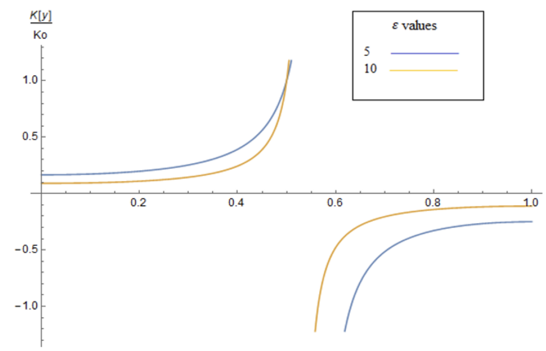

Fig. 3. Plot of $K(y)/K_o$ relationship with values of ε.

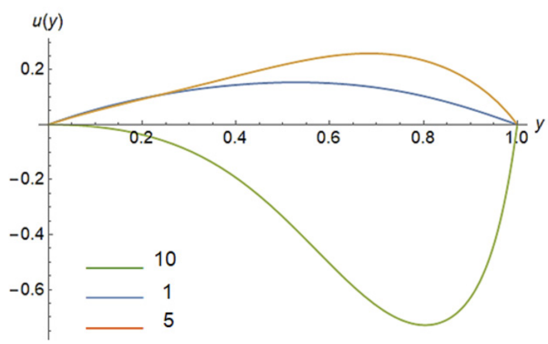

Fig. 4. Plot of *u(y)* with variation of $\varepsilon = \{1, 5, 10\}$ for constant values of $M = 1$, $Da = 0.1$, $\phi = 0.35$, and $P = 5$.

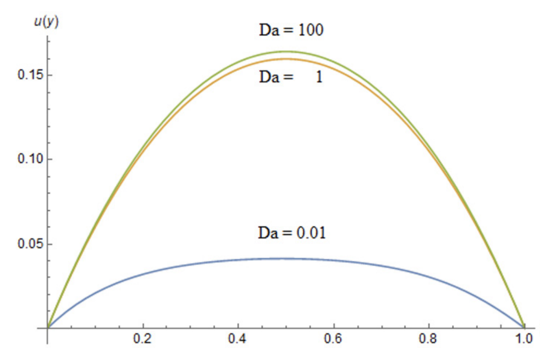

Fig. 5. Plot of *u(y)* with variation of *Da* for constant values of $M = 3$, $\phi = 0.35$, $P = 5$ and $\varepsilon = 0.01$

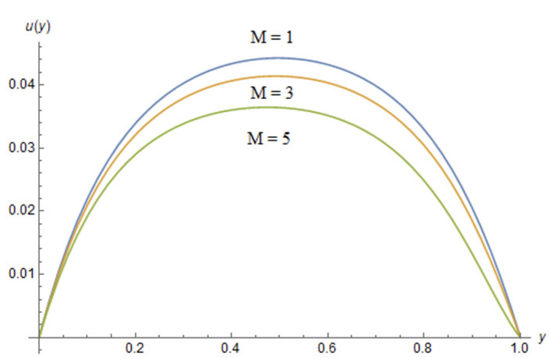

Fig. 6. Plot of *u(y)* with variation of *M* with constant values of $Da = 0.01$, $\phi = 0.35$, $P = 5$ and $\varepsilon = 0.01$

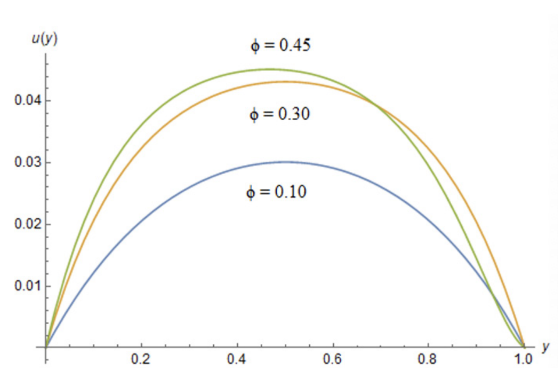

Fig. 7. Plot of *u(y)* with variation of ϕ with constant values of $Da = 0.01$, $M = 0.35$, $P = 5$ and ε=0.01

The second part of the results was divided into three sections for a better analysis of non-dimensional volumetric flow Q with concerning flow parameters. To do this, we first proceed to plot Q in function of Da and M. In Fig. 8 the values of porosity, pressure gradient, and amplitude of the permeability variation have been taken as constants, as $\phi = 0.35$, $P = 5$, and $\varepsilon = 0.01$, respectively. Under these parameters, the value of Q grows very fast from the minimum, in the range of $0.01 < Da < 1$, getting the maximum value when M is zero. As the value of M grows, the value of Q decreases sharply. Notice the good behavior of the Q curve in the range of $0.01 < Da < 1$, which corresponds to the Brinkman flow regime.

In Fig. 9, on the other hand, Q is taken as a function of ε and ϕ, taking as constant the values of *P, M*, and *Da* at 5, 3, and 0.01, respectively. The maximum values of Q are concentrated in the region included in the coordinates $0.5 < \varepsilon < 1$ and $0.15 < \phi < 0.45$. The characteristic value taken for the porosity $\phi = 0.35$ is based on this evidence throughout this article.

In Fig. 10, finally, Q has been taken as a function of *Da* and ϕ, taking as constant the values of *P, M* and ε at 5, 3, and 0.01, respectively. In the range of $0.01 < Da < 1$ the value of Q grows very fast and its growth concerning porosity is uniform, almost linearly increasing.

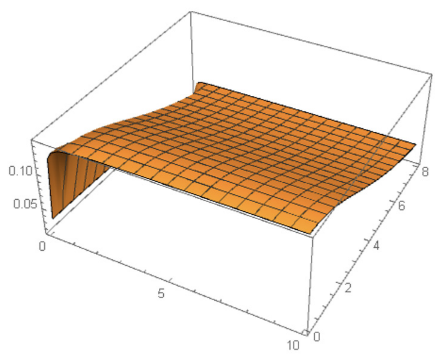

Fig. 8. 3D Plot for *Q* with variation of $Da = \{0.01, 10\}$ and $M = \{0, 8\}$ with constant values of $\phi = 0.35$, $P = 5$, and $\varepsilon = 0.01$

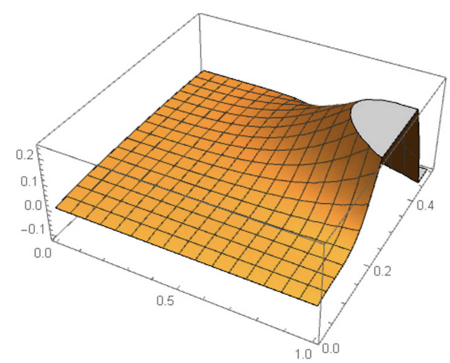

Fig. 9. 3D Plot for *Q* with variation of $\varepsilon = \{0.001, 1\}$ and $\phi = \{0.01, 0.5\}$ with constant values of $P = 5$, $M = 3$, and $Da = 0.01$.

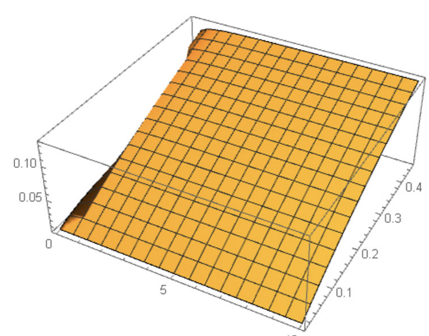

Fig. 10. 3D Plot for *Q* with variation of $Da = \{0.01, 10\}$ and $\phi = \{0.01, 0.45\}$ with constant values of $P = 5$, $M = 3$, and $\varepsilon = 0.01$.

The third part consists of the analysis of results obtained through the shear stress at the upper and bottom plates. The shear stress τ_w results are presented in Table 3. When the magnetic number M is increased, τ_w decrease at the bottom plate and inversely in the upper plate, maintaining the other parameters constant.

Table 3. Non-dimensional shear stress at the lower and upper plates

	y = 0	**y = 1**
M = 0, ε = Da = 0.01, ϕ = 0.35	0.29314558371923	-0.25403611914033
M = 1, ε = Da = 0.01, ϕ = 0.35	0.29173600000000	-0.24991200000000
M = 2, ε = Da = 0.01, ϕ = 0.35	0.28762000000000	-0.23675200000000
M = 3, ε = Da = 0.01, ϕ = 0.35	0.28111300000000	-0.21175700000000

The fourth part consists of to compare the results obtained by the numerical method and PSM for validation. The numerical results were obtained by the Shooting Method using Mathematica. For the specific values of M, Da, ϕ, P, and ε, the Table 4 permits to show the difference between the two methods with $O(y^{16})$ and $O(y^{21})$. With a higher order of power in the PSM, the difference is less respected than the numerical method (NM).

Table 4. Comparison values of $u(y)$ by NM and PSM

M=1, ε=Da=0.01, ϕ=0.35, P=5

y	NM	PSM $O(y^{16})$	PSM $O(y^{21})$	Difference $O(y^{21})$	Difference $O(y^{21})$
0.1	0.0219417000	0.0219270000	0.0219417849	1.470E-05	8.490E-08
0.2	0.0339204000	0.0338856000	0.0339205413	3.480E-05	1.413E-07
0.3	0.0403217000	0.0402538000	0.0403218671	6.790E-05	1.671E-07
0.4	0.0434633000	0.0433378000	0.0434637158	1.255E-04	4.158E-07
0.5	0.0444531000	0.0442240000	0.0444538122	2.291E-04	7.122E-07
0.6	0.0436038000	0.0431890000	0.0436050937	4.148E-04	1.293E-06
0.7	0.0405587000	0.0398205000	0.0405609471	7.382E-04	2.247E-06
0.8	0.0341727000	0.0329402000	0.0341767386	1.2325E-03	4.038E-06
0.9	0.0221087000	0.0204891000	0.0221147011	1.6196E-03	6.001E-06

VI. Conclusions

The modified model of Brinkman magnetorheological fully developed fluid flow in a porous medium with periodic permeability proposed by Mathew and non-slip condition, including the Ochoa-Tapia and Whitaker effective viscosity equation was solved. For the analysis, the problem was divided into four parts. Each one studied a special characteristic of the problem of flow in the porous medium, trying to visualize to the greatest extent possible the behavior of important physical quantities such as velocity, volumetric flow, and the shear stress on the walls. It can be observed that incorporating the model of the effective viscosity, the maximum value of porosity stability is 0.45, unlike the Mathew model that considered the unit viscosity ratio.

It has also been emphasized that the amplitude ε must be much less than 1. The solution method was selected considering the stability and speed of convergence of the validated results. As it could be seen, the greater the degree of the power series, the response is closer to the results obtained by the numerical method, which validates the response achieved by this method, which is widely known.

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VIII. References

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